Math 60 10.3-2nd and 10.4-1st

10.3 Solving Equations Quadratic in Form - 2nd
10.4 Graphing Quadratic Functions Using Transformations. - 1st

10.3
objectives: additional practice

10.4
objectives:

1) Graph quadratic functions
(parabolas opening up or down)
using transformations, also known as
changes to the basic parabola $f(x)=x^2$

- $f(x)=x^2+k$ vertical shift k units
- $f(x)=(x-h)^2$ horizontal shift h units

to be continued in next lesson.

Math 60 10.3 - 1st and 10.4 - 2nd

① Solve $n^{2/3} - 4n^{1/3} - 21 = 0$

Review from yesterday:
<u>Basic steps</u> for solving an equation that's not quadratic but is quadratic in form.

1) Recognize quadratic form $\quad u = \text{stuff}$, $u = (\text{stuff})^2$ — same stuff
2) Substitute $u = \text{stuff}$, $u^2 = (\text{stuff})^2$ to get a true quadratic.
3) Solve by any quadratic method
   - square root property
   - factoring
   - completing the square (CTS)
   - quadratic formula (QF)
4) Replace u by stuff again, in both solutions
5) Isolate the variable in each equation.

Solve $n^{2/3} - 4n^{1/3} - 21 = 0$

$$\begin{cases} u = n^{1/3} \\ u^2 = n^{2/3} \end{cases}$$

$$u^2 - 4u - 21 = 0$$
$$(u-7)(u+3) = 0$$
$$u = 7 \qquad u = -3$$
$$n^{1/3} = 7 \qquad n^{1/3} = -3$$
$$(n^{1/3})^3 = 7^3 \qquad (n^{1/3})^3 = (-3)^3$$
$$\boxed{n = 343} \qquad \boxed{n = -27}$$

Math 60 10.3-1st and 10.4-2nd

② Solve $c^{1/2}+c^{1/4}-12=0$

Is $u=c^{1/2}$ or is $u=c^{1/4}$? check by finding u^2.

$$u^2=(c^{1/2})^2$$
$$u^2=c^{1/2\cdot 2}$$
$$u^2=c$$
$\Uparrow$
There is no c in the equation
This is wrong.

$$u^2=(c^{1/4})^2$$
$$u^2=c^{1/4\cdot 2}$$
$$u^2=c^{1/2}$$
$\Uparrow$
There is a $c^{1/2}$ in the equation.
This is correct

$u=c^{1/4}$
$u^2=c^{1/2}$

$$u^2+u-12=0$$
$$(u+4)(u-3)=0$$

$u=-4$	$u=3$
$c^{1/4}=-4$	$c^{1/4}=3$
$\sqrt[4]{c}=-4$	$\sqrt[4]{c}=3$
$(\sqrt[4]{c})^4=(-4)^4$	$(\sqrt[4]{c})^4=3^4$
$c=256$	$c=81$

Are you wide awake?
$\sqrt[4]{c}=-4$
can't happen!
n is an even index.

$$c^{1/2}+c^{1/4}-12=0$$
$$\sqrt{c}+\sqrt[4]{c}-12=0$$

We raised both sides to an even power,

so MUST check for extraneous.

$c=256$ $\quad \sqrt{256}+\sqrt[4]{256}-12\stackrel{?}{=}0$
$$16+4-12=0$$
$$20-12=0$$
$$8\neq 0 \quad \underline{\text{no}}.$$

$c=81$ $\quad \sqrt{81}+\sqrt[4]{81}-12\stackrel{?}{=}0$
$$9+3-12=0 \checkmark$$

$$\boxed{c=81}$$

Math 60 10.3 - 1st and 10.4 - 2nd

(3) Solve $y^6 - 7y^3 - 8 = 0$ for all real and complex solutions.

Note: degree 6! Expect 6 solutions.

$u = y^3$

$u^2 = (y^3)^2 = y^6$

$$u^2 - 7u - 8 = 0$$

$$(u-8)(u+1) = 0$$

$$u = 8 \qquad u = -1$$

$$y^3 = 8 \qquad y^3 = -1$$

We can't just take cube roots because we want three solutions for each, a total of six solutions.

$$y^3 - 8 = 0 \qquad y^3 + 1 = 0$$

Set $= 0$.

$$(y-2)(y^2 + 2y + 4) = 0 \qquad (y+1)(y^2 - y + 1) = 0$$

factor sum of cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$\boxed{y = 2}$ $\qquad$ $\boxed{y = -1}$

QF

factor diff of cubes

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

S O AP

$$y = \frac{-2 \pm \sqrt{4 - \cancel{4}(1)(4)}}{2}$$

$$y = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$y = \frac{-2 \pm \sqrt{-12}}{2}$$

$$y = \frac{-2}{2} \pm \frac{2i\sqrt{3}}{2}$$

$$\boxed{y = -1 \pm i\sqrt{3}}$$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$y = \frac{1 \pm \sqrt{1-4}}{2}$$

$$y = \frac{1 \pm \sqrt{-3}}{2}$$

$$\boxed{y = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}}$$

Yes, six solutions:

$$\boxed{y = 2,\ -1 + i\sqrt{3},\ -1 - i\sqrt{3},\ -1,\ \frac{1}{2} + \frac{i\sqrt{3}}{2},\ \frac{1}{2} - \frac{i\sqrt{3}}{2}}$$

Math 60 10.3 – 2nd and 10.4 – 1st

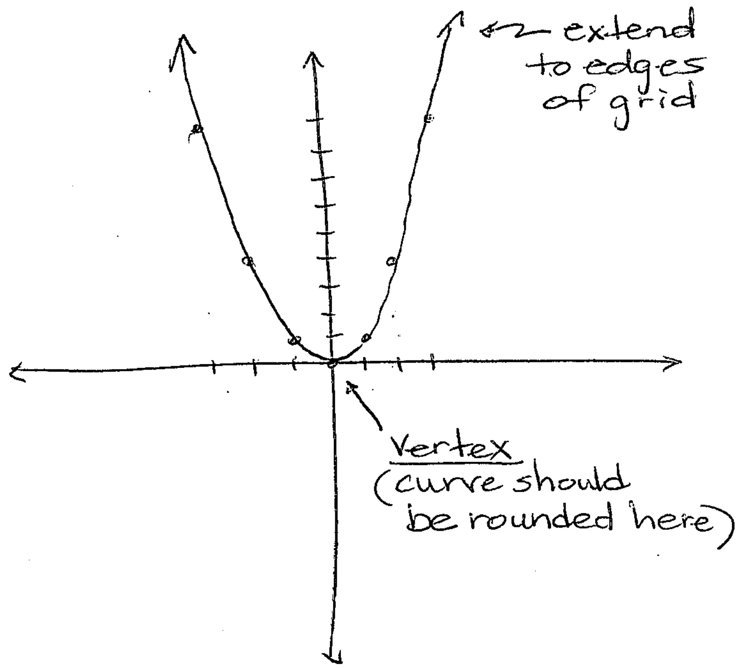
Recall: Graph of basic quadratic function

$f(x) = x^2$

x	y
0	0
1	1
2	4
3	9
4	16
−1	1
−2	4
−3	9
−4	16

perfect square numbers

same #s on leftside of graph



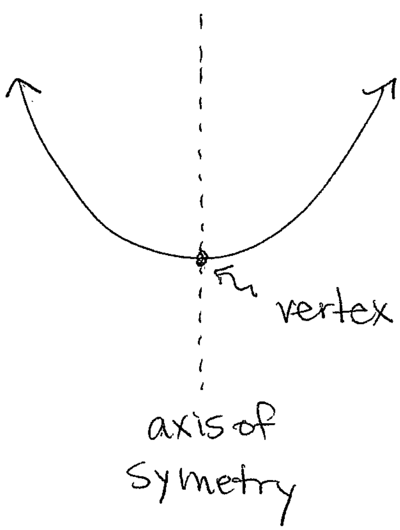
Math 60 10.4

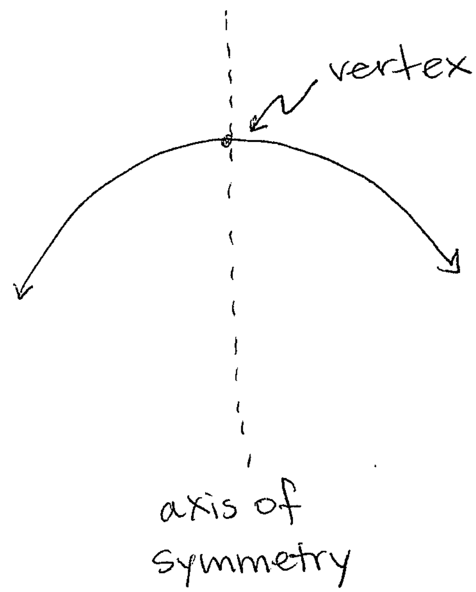
Remember also that the left and right sides of the graph have the same shape.

This is called symmetry across a line.

The line which divides the graph into two mirror images is called the axis of symmetry.

The axis of symmetry always passes through the vertex, which is the lowest point on an upward-opening parabola (or the highest point on a downward-opening parabola).





The axis of symmetry for a quadratic function (passes V.L.T.) is a vertical line. Its equation is $\boxed{x=h}$, where h is the x-coordinate of the vertex.

{vertical line equations are always $x = \#$}

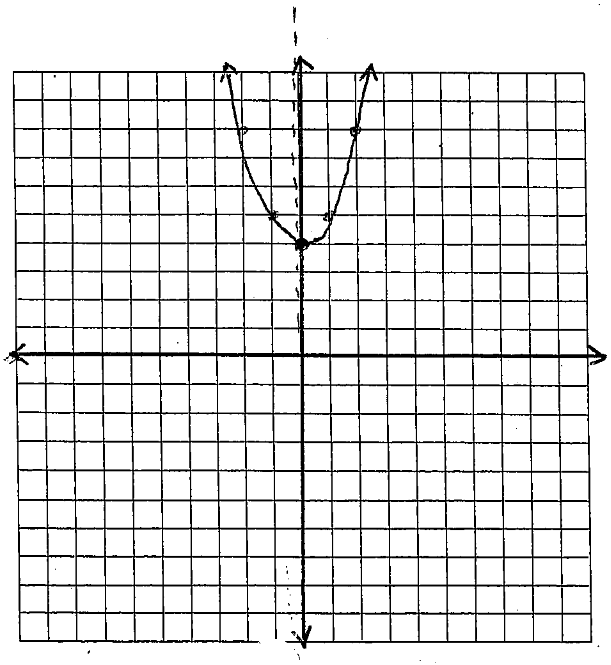
Math 60 10.4

Graph each function.

(4) $f(x) = x^2 + 4$

x	y	
0	4	0^2+4
1	5	1^2+4
2	8	2^2+4
3	13	3^2+4
4	20	4^2+4
−1	5	$(-1)^2+4$
−2	8	repeat
−3	13	
−4	20	

axis of symmetry $x=0$



This is the basic $y=x^2$ graph, shifted up 4 units.

$f(x) = x^2 + k$

means shift the basic parabola $y=x^2$ k units up

$f(x) = x^2 - k$

means shift the basic parabola $y=x^2$ k units down

Notice that k is <u>not</u> inside parentheses, and is not squared.

Math 60 10.4

(5) $f(x) = (x-2)^2$

x	y
0	4
1	1
2	0
3	1
4	4
−1	9
−2	16
−3	25
−4	36
+5	9

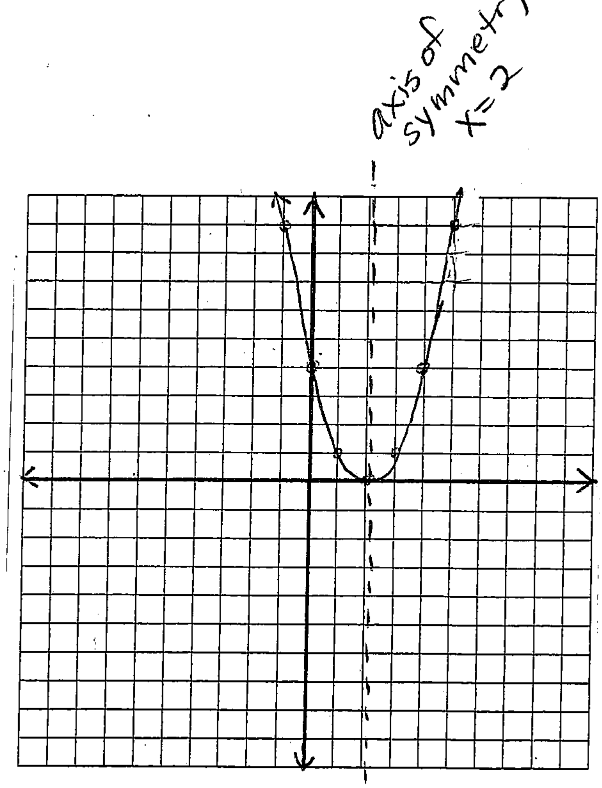
$(0-2)^2 = (-2)^2 = 4$

$(1-2)^2 = (-1)^2 = 1$

$(-1-2)^2 = (-3)^2 = 9$

$\left.\begin{matrix} 16 \\ 25 \\ 36 \end{matrix}\right\}$ not helpful!

$9 \}$ more helpful!



This is our basic parabola $y=x^2$, but shifted right 2 units.

$f(x) = (x-h)^2$

means shift the basic parabola $y=x^2$ h units to the right

CAUTION This may seem counter-intuitive.
right is usually positive,
but translation right happens for subtracted h.

$f(x) = (x+h)^2$

means shift the basic parabola $y=x^2$ h units to the left

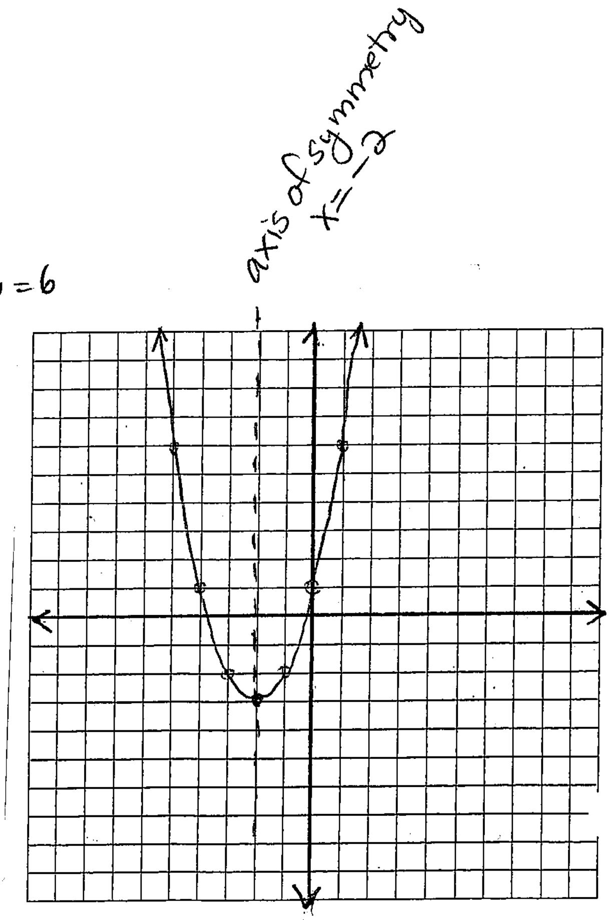
Notice that h is inside parentheses and is squared.

Math 60 10.4

6) $f(x) = (x+2)^2 - 3$

Option 1: The long way.

x	y		
0	1		$(0+2)^2-3 = 4-3 = 1$
1	6		$(1+2)^2-3 = 3^2-3 = 9-3 = 6$
2	13	not helpful	$(2+2)^2-3 = 16-3 = 13$
3	22	not helpful	$(3+2)^2-3 = 22$
4	33	not helpful	$(4+2)^2-3 = 33$
-1	-2		$(-1+2)^2-3 = -2$
-2	-3		$(-2+2)^2-3 = -3$
-3	-2	repeating values ☺	$(-3+2)^2-3 = -2$
-4	1	repeating values ☺	$(-4+2)^2-3 = 1$
-5	6	repeating values ☺	$(-5+2)^2-3 = 6$
-6	13	repeating values ☺	$(-6+2)^2-3 = 13$



Option 2: Notice $f(x) = (x+2)^2 - 3$ means

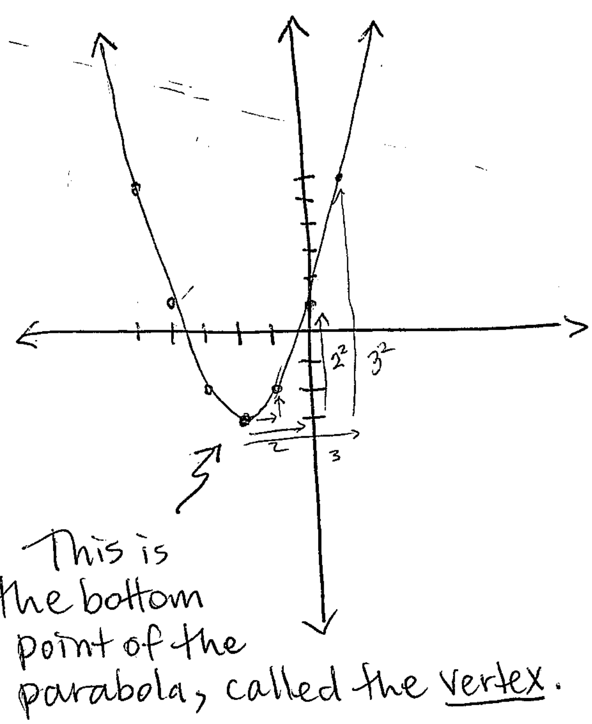
$(x+2)$: shifted left 2 units

-3: shifted down 3 units.

Use pattern of squares, starting from the vertex:

right 1, up 1^2
right 2, up 2^2
right 3, up 3^2

left 1, up 1^2
left 2, up 2^2
left 3, up 3^2



This is the bottom point of the parabola, called the vertex.